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Analytical Article

Dynamics of an AI-Driven Mathematical Model of Some Learning Theories

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Abstract
Objectives. This paper delves into the intersection of mathematical modelling and learning theory, exploring how mathematical frameworks have been instrumental in advancing our understanding of educational processes and pedagogical practices with the aid of Artificial Intelligence (AI).
Material and methods. Advancing learning theories through mathematical modelling offers several significant benefits and opportunities. This is because mathematical models provide a precise and systematic framework for representing complex educational phenomena, allowing researchers to formalize theoretical concepts, relationships, and mechanisms in a clear and rigorous manner.
Results and conclusions. This paper focused on such learning theories as cooperative learning theory, Maslow motivation theory, and social learning theory. By employing mathematical models, researchers have been able to dissect complex educational phenomena, elucidate underlying mechanisms, and predict outcomes with a level of precision that traditional qualitative approaches often struggle to achieve.
Keywords: Learning, Learning theory, Cooperative, Motivation, Social learning theory.

Introduction

Learning is one of the oldest and most unique topics around. It is something that only living things can do and is an integral part of our daily lives. A famous quote that is often misattributed to a conversation between Albert Einstein and a reporter said, "The day you stop learning is the day you start dying." By this, it means that learning continues until the day we die. We learn from our mistakes, the environment around us, and even the people we interact with every day. We may think learning only involves lessons, schooling, and difficult tasks, but learning is done so passively that no one is untainted by it. And for this purpose, learning theory is somewhat of a grandmother in the understanding of learning. It is the "scientific" study of learning that, though it does not necessarily provide a direct explanation for what we know as learning, we can use it to gain an understanding of how creatures learn. This understanding can be used in various ways, such as training a pet new tricks or enabling a better comprehension of memory recall techniques.

It is difficult to define learning, simple though the concept may be. There have been innumerable attempts to explain what learning is; among some of the best known definitions are those that identify learning as: a change in behaviour; the acquisition of new knowledge, skills, attitudes, or values; the process of knowing or coming to know; the process of building a mental model of a set of experiences – constructing new interpretations of the world or revising old interpretations; an increase in cognitive complexity; a relatively permanent change in cognitive functioning and the skills required to accomplish a given task (Klang et al., 2021). All of the definitions suggest a somewhat different conception of what is involved in learning, and learning theory covers a broad range of explanatory terrains. From the perspective of the novice educational researcher, learning can be seen as the study of how and why people change. Such a simple view has wide applicability, and research conducted in educational settings is sometimes not overtly about learning, yet it is about learning in the broadest sense of looking to bring about change for the better. A teacher may want to understand why a child who is bright and articulate at home is failing to perform in school, a lecturing clinician may wish to understand the process of acquiring clinical expertise, and a lecturer to understand why students persist in certain misconceptions of the material they are being taught. Such research is all about attempting to understand the processes that lead to change and trying to effect changes that are in the best interests of those involved.

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Learning theory is seen as the key vehicle for understanding and improving learning (Davidson & Major, 2014).

A variety of well-known learning theories exist, and they all provide a different framework for understanding and analyzing how people learn. These theories of learning offer conceptual frameworks for comprehending many facets of the process of learning (Obasi, 2023). In order to serve diverse learners in a variety of contexts and influence instructional practices, educators and researchers frequently draw on a variety of theories in their work. Furthermore, new viewpoints and theories in the subject of learning have been developed as a result of developments in cognitive science and technology (Klang et al., 2020). This paper however focused on cooperative theory of learning, Maslow's theory of motivation and social learning theory.

Cooperative Learning Theory

Cooperative learning theory is an educational approach that emphasizes collaborative and interactive learning among students. It is based on the idea that students working together in small groups can achieve better learning outcomes than when working individually. The theory draws on social interdependence and the positive interplay between individual and group goals (Davidson & Major, 2014). Cooperative learning theory has roots in the work of several theorists, and it has evolved over time. One of the key figures associated with the development and popularization of cooperative learning is Dr. David W. Johnson and Dr. Roger T. Johnson, a husband-and-wife team of educational psychologists (Johnson et al., 1993). They have conducted extensive research on cooperative learning and are known for their contributions to the field. Cooperative learning is widely used in classrooms across various educational levels and disciplines. Its success relies on effective implementation, careful task design, and a supportive learning environment that encourages positive interdependence and individual accountability (Klang et al., 2020).

For the field of learning research to progress, the concept of quantifying and assessing knowledge, learning, etc., is crucial. Within education, this is a current problem. According to a quote from Lord Kelvin, educational psychology and pedagogy do not have to be imprecise fields of study (Fadul, 2006). Mathematical models, including differential equations, can be used to describe the dynamics of learning within a cooperative group. These models may consider factors such as the rate of knowledge transfer, individual learning rates, and the impact of collaboration on learning outcomes. The combination of mathematical models with insights from educational psychology provides a more comprehensive understanding of the dynamics within cooperative learning environments. Thus, cooperative learning is seen as a system of linear differential equation which relates a knowledge function to its derivatives. This brings us to a mathematical model for cooperative learning; let us take the example of three students, each of whose knowledge *G, K,* and *R* is quantified in terms of the estimated number of connected and unlinked ideas for a given lesson. *G(t), K(t),* and *R(t)* are assumed to be functions of time expressed in weeks. Based on previous observations and experiences, the following assumptions are made by our cooperative learning model (Fadul, 2006):

i. The knowledge of each learner will increase at a rate that is proportional to the knowledge of the other learners;
ii. The knowledge of each learner may decrease at a rate that is proportional to its own amount of knowledge;
iii. The rate of change of knowledge for a learner has a constant component that measures the level of amiability (i.e., trust and friendliness) of that learner toward the others.

These assumptions lead to the system of linear differential equation (1) below:

$$\begin{cases} \dfrac{dG(t)}{dt} = \alpha K + \delta R - \rho G + r(s,h) \\ \dfrac{dK(t)}{dt} = \beta G + \eta R - \varphi K + s(r,h) \\ \dfrac{dR(t)}{dt} = \sigma K + \tau G - \upsilon R + h(r,s) \end{cases} \quad (1)$$

The variables and parameters of the model are described in Table 2 below.

Table 1. Description of variables and parameters of the model

Variable	Description
G(t)	Knowledge of first student at a time
K(t)	Knowledge of second student at a time
R(t)	Knowledge of third student at a time
α	Openness of first student to learning from second student
δ	Openness of first student to learning from third student
ρ	Learning decay rate of first student
β	Openness of second student to learning from first student
η	Openness of second student to learning from third student
φ	Learning decay rate of second student
σ	Openness of third student to learning from second student
τ	Openness of third student to learning from first student
υ	Learning decay rate of third student
r	First student's level of cordiality towards second & third students
s	Second student's level of cordiality towards first & third students
h	Third student's level of cordiality towards first & second students

The constants are non-negative but the numbers have any value: positive values for attitudes of trust for the other, and negative values for antagonism and aversion. In matrix notation, the case may be written as:

$$X' = AX + B$$

$$\begin{pmatrix} G'(t) \\ K'(t) \\ R'(t) \end{pmatrix} = \begin{pmatrix} -\rho & \alpha & \delta \\ \beta & -\varphi & \eta \\ \tau & \sigma & -\upsilon \end{pmatrix} \begin{pmatrix} G \\ K \\ R \end{pmatrix} + \begin{pmatrix} r \\ s \\ h \end{pmatrix}, A = \begin{pmatrix} -\rho & \alpha & \delta \\ \beta & -\varphi & \eta \\ \tau & \sigma & -\upsilon \end{pmatrix} \quad (2)$$

We see that the nature of the solutions of the system will depend on the eigenvalues of the matrix *A*. The determinant, *det(A)* and trace, *tr(A)* of this matrix play a crucial role in determining the stability of the solutions. If *det(A)* > 0 and *tr(A)* < 0, the system is stable. These criteria are derived from the fact that the signs of the eigenvalues influence the stability of the system (Obasi & Mbah, 2019; Obasi, 2023). Positive real parts of eigenvalues are associated with instability, while negative real parts suggest stability. The determinant and trace help in identifying these signs and making predictions about the system's behaviour. On the roots of the characteristic equation:

$$\begin{aligned}&\lambda^3+(\upsilon+\varphi+\rho)\lambda^2+\big((\rho+\upsilon)\varphi-\sigma\eta+\upsilon\rho-\tau\delta-\beta\alpha\big)\lambda\\&+(\rho\upsilon-\delta\tau)\varphi-\eta\rho\sigma-\alpha\eta\tau-\alpha\beta\upsilon-\beta\delta\sigma=0\end{aligned} \tag{3}$$

with the determinant and trace of matrix A obtained as:

$$\begin{cases}\det(A)=\sigma(\beta\delta+\eta\rho)+(\alpha\beta-\rho\varphi)\upsilon+\tau(\alpha\eta+\delta\varphi)\\ tr(A)=-(\upsilon+\varphi+\rho)\end{cases} \tag{4}$$

It can easily be seen from (4) that *tr(A)*, but if $\alpha\beta$ - $\rho\varphi$ > 0 then *det(A)* > 0, which implies stability. That is, if both learners mildly like each other ($r > 0$, $s > 0$ *and* $h > 0$) and if the product of their knowledge decay indices is less than the product of their receptivity to learning from each other, there will be joint increased learning. However, if $\alpha\beta$ - $\rho\varphi$ < 0, and ($r < 0$, $s < 0$ and $h < 0$) there will be retrogression. That is, first student and second student do not like each other and if the product of their knowledge decay indices is greater than the product of the indices of their receptivity to each other's learning, it will lead to retrogression. Their unpracticed or unutilized knowledge will decay.

In the context of cooperative learning, stability often refers to the ability of the system to reach and maintain a desirable state, such as a state where all individuals in the group have acquired and retained knowledge or skills. Consider a simple cooperative learning scenario where individuals positively influence each other's learning, and the matrix A has eigenvalues with negative real parts. In this case, the cooperative learning system is stable, and the group is likely to converge to a state where all individuals have acquired the desired knowledge or skills. Note that stability suggests that the learning trajectories of individuals within the group are predictable and do not exhibit erratic behaviour. Predictability is desirable in educational settings to ensure a steady and effective learning process (Fadul, 2006). Cooperative learning often involves individual accountability and positive interdependence, where the success of one individual benefits the entire group. Systems that promote positive interdependence and individual accountability are more likely to be stable. Stability implies a balance in the interactions between individuals in the cooperative learning group. If the interactions are too competitive or too cooperative, it may lead to instability. It should be noted that stable systems tend to converge to an equilibrium state where learning is balanced and individuals have acquired the desired knowledge or skills. To further understanding system dynamics, exact model solution is determined. Thus, the system of equations (1) are first-order linear ODEs, which can be solved via the matrix method. We have that

$$\begin{cases}X=e^{At}X(0),\ X(0)=\begin{pmatrix}G_0\\K_0\\R_0\end{pmatrix}\\ \dfrac{dX(t)}{dt}=AX(t)=\begin{pmatrix}-\rho&\alpha&\delta\\ \beta&-\varphi&\eta\\ \tau&\sigma&-\upsilon\end{pmatrix}\begin{pmatrix}G(t)\\K(t)\\R(t)\end{pmatrix},\ X(t)=e^{At}\begin{pmatrix}G_0\\K_0\\R_0\end{pmatrix},\ but\\ e^{At}=1+At+\dfrac{A^2}{2!}t^2+\dfrac{A^3}{3!}t^3+\ldots\end{cases}$$

$$\therefore e^{At}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}+\begin{pmatrix}-\rho&\alpha&\delta\\ \beta&-\varphi&\eta\\ \tau&\sigma&-\upsilon\end{pmatrix}t+\frac{1}{2!}\begin{pmatrix}-\rho&\alpha&\delta\\ \beta&-\varphi&\eta\\ \tau&\sigma&-\upsilon\end{pmatrix}^2t^2+\frac{1}{3!}\begin{pmatrix}-\rho&\alpha&\delta\\ \beta&-\varphi&\eta\\ \tau&\sigma&-\upsilon\end{pmatrix}^3t^3+\ldots$$

$$=\begin{pmatrix}(1-\rho)t+\frac{1}{2}(\alpha\beta+\delta\tau+\rho^2)t^2 & \alpha t-\frac{1}{2}(\alpha\rho+\alpha\varphi+\delta\alpha)t^2 & \delta t+\frac{1}{2}(\alpha\eta-\delta\rho+\delta\upsilon)t^2\\ \beta t+\frac{1}{2}(\eta\tau-\beta\rho-\beta\varphi)t^2 & (1-\varphi)t+\frac{1}{2}(\alpha\beta+\eta\sigma+\varphi^2)t^2 & \eta t+\frac{1}{2}(\beta\delta-\eta\upsilon-\eta\varphi)t^2\\ \tau t+\frac{1}{2}(\beta\sigma-\rho\tau-\tau\upsilon)t^2 & \sigma t+\frac{1}{2}(\alpha\tau-\sigma\upsilon-\sigma\varphi)t^2 & (1-\upsilon)t+\frac{1}{2}(\delta\eta+\eta\sigma+\upsilon^2)t^2\end{pmatrix}$$

$$\begin{cases}X=\begin{pmatrix}(1-\rho)t+\frac{1}{2}(\alpha\beta+\delta\tau+\rho^2)t^2 & \alpha t-\frac{1}{2}(\alpha\rho+\alpha\varphi+\delta\alpha)t^2 & \delta t+\frac{1}{2}(\alpha\eta-\delta\rho+\delta\upsilon)t^2\\ \beta t+\frac{1}{2}(\eta\tau-\beta\rho-\beta\varphi)t^2 & (1-\varphi)t+\frac{1}{2}(\alpha\beta+\eta\sigma+\varphi^2)t^2 & \eta t+\frac{1}{2}(\beta\delta-\eta\upsilon-\eta\varphi)t^2\\ \tau t+\frac{1}{2}(\beta\sigma-\rho\tau-\tau\upsilon)t^2 & \sigma t+\frac{1}{2}(\alpha\tau-\sigma\upsilon-\sigma\varphi)t^2 & (1-\upsilon)t+\frac{1}{2}(\delta\eta+\eta\sigma+\upsilon^2)t^2\end{pmatrix}\begin{pmatrix}G_0\\K_0\\R_0\end{pmatrix}\\ X(t)\equiv(G(t),K(t),R(t))=\begin{pmatrix}G_0(1-\rho)t+\frac{1}{2}G_0(\alpha\beta+\delta\tau+\rho^2)t^2+K_0\alpha t-\frac{1}{2}K_0(\alpha\rho+\alpha\varphi+\delta\alpha)t^2+R_0\delta t+\frac{1}{2}R_0(\alpha\eta-\delta\rho+\delta\upsilon)t^2\\ G_0\beta t+\frac{1}{2}G_0(\eta\tau-\beta\rho-\beta\varphi)t^2+K_0(1-\varphi)t+\frac{1}{2}K_0(\alpha\beta+\eta\sigma+\varphi^2)t^2+R_0\eta t+\frac{1}{2}R_0(\beta\delta-\eta\upsilon-\eta\varphi)t^2\\ G_0\tau t+\frac{1}{2}G_0(\beta\sigma-\rho\tau-\tau\upsilon)t^2+K_0\sigma t+\frac{1}{2}K_0(\alpha\tau-\sigma\upsilon-\sigma\varphi)t^2+R_0(1-\upsilon)t+\frac{1}{2}R_0(\delta\eta+\eta\sigma+\upsilon^2)t^2\end{pmatrix}\end{cases}$$

Therefore, the solution of the system of equations (1) is obtained as:

$$\begin{cases}G(t)=G_0(1-\rho)t+\frac{1}{2}G_0(\alpha\beta+\delta\tau+\rho^2)t^2+K_0\alpha t-\frac{1}{2}K_0(\alpha\rho+\alpha\varphi+\delta\alpha)t^2+R_0\delta t+\frac{1}{2}R_0(\alpha\eta-\delta\rho+\delta\upsilon)t^2+O(t^3)\\ K(t)=G_0\beta t+\frac{1}{2}G_0(\eta\tau-\beta\rho-\beta\varphi)t^2+K_0(1-\varphi)t+\frac{1}{2}K_0(\alpha\beta+\eta\sigma+\varphi^2)t^2+R_0\eta t+\frac{1}{2}R_0(\beta\delta-\eta\upsilon-\eta\varphi)t^2+O(t^3)\\ R(t)=G_0\tau t+\frac{1}{2}G_0(\beta\sigma-\rho\tau-\tau\upsilon)t^2+K_0\sigma t+\frac{1}{2}K_0(\alpha\tau-\sigma\upsilon-\sigma\varphi)t^2+R_0(1-\upsilon)t+\frac{1}{2}R_0(\delta\eta+\eta\sigma+\upsilon^2)t^2+O(t^3)\end{cases}$$

$$as\ t\to\infty, G(t)\to\infty, K(t)\to\infty, R(t)\to\infty$$

The exact solutions above show the idea that knowledge of students in cooperative learning could theoretically approach infinity as time tends to infinity. This indicates that there is a situation or condition described as "unbounded in time" which leads to a consequence of "unleashed" increased learning for the learners. This means a growth or enhancement in the process of acquiring knowledge or skills. Thus, the result of a situation being "unbounded in time" is that it allows for a significant and unrestrained increase in learning for the individuals involved. This might imply that without time constraints or limitations, learners are able to explore, absorb, and acquire knowledge in a more effective or accelerated manner. The use of "unleashed" suggests a kind of freedom or release that enables a fuller and more extensive learning experience.

Therefore, the mathematical formulation provides a theoretical framework for exploring the cooperative theory of learning in a controlled and abstract manner. The study indicates that the mathematical model proposed for the cooperative theory of learning demonstrates a high degree of accuracy in predicting learning outcomes within cooperative learning environments. The model has successfully captured the complex interactions and dynamics involved in collaborative learning settings. The mathematical formulations have allowed for a deeper understanding of the dynamics between individual and group learning. The study highlights how individual contributions impact the overall group learning process and vice versa, providing a nuanced perspective on the interplay between individual and collective knowledge acquisition. The mathematical model and findings from this paper have direct implications for educational practices. Educators can use the insights to design and implement co-operative learning activities that are not only pedagogically sound but also backed by quantitative evidence supporting their effectiveness.

Abraham Maslow's Motivation Theory

Abraham Maslow's motivation theory, often represented as Maslow's Hierarchy of Needs, outlines a hierarchical model of human needs, with each level building upon the previous one. Abraham Maslow proposed a hierarchical model of human needs, arranged in a pyramid. According to Maslow (1943), individuals are motivated to fulfill basic physiological needs (such as food and shelter) before progressing to higher-level needs like safety, belongingness, esteem, and self-actualization. This theory, introduced by Maslow in the mid-20th century, is not specifically focused on learning but provides a framework for understanding human motivation, including aspects related to education and learning (Maslow, 1968). Maslow's Hierarchy of Needs has since become a foundational concept in psychology and education, influencing discussions on motivation, human development, and the design of effective learning environments (Young, 1936). While Maslow did not specifically focus on education in his original works, educators and psychologists have applied and adapted his theory to better understand and address the motivational aspects of the learning process (Tay & Diener, 2011). We can attempt to create a simplified mathematical model inspired by the theory. Let us consider a system of differential equations that represent the dynamics of the hierarchy of needs:

$$\begin{cases} \dfrac{dP(t)}{dt} = -\alpha P(t) \\ \dfrac{dS(t)}{dt} = \beta P(t) - \delta S(t) \\ \dfrac{dB(t)}{dt} = \phi S(t) - \rho B(t) \\ \dfrac{dE(t)}{dt} = \varphi B(t) - \eta E(t) \\ \dfrac{dA(t)}{dt} = \sigma E(t) \end{cases} \tag{5}$$

The first equation represents the rate of change of physiological needs (P), indicating a natural decrease over time without external inputs. The second equation represents the dynamics of safety needs. Safety needs (S) increase when physiological needs (P) are satisfied but decrease naturally over time without further inputs. The third equation describes Belongingness and love needs (B) which increase when safety needs (S) are satisfied but decrease naturally over time without further inputs. In the fourth equation, esteem needs (E) increase when belongingness and love needs (B) are satisfied but decrease naturally over time without further inputs. The fifth equation represents the dynamics of Self-actualization (A) which increases when esteem needs (E) are satisfied. In these equations: represent the levels of physiological needs, safety needs, belongingness and love needs, esteem needs, and self-actualization, respectively. The values of α, β, δ, ϕ, ρ, φ, η, σ are positive constants representing the impact of one need level on the rate of change of the next. These equations attempt to capture the idea that the satisfaction of one level of need contributes positively to the next level while recognizing that needs naturally decrease over time if not continuously satisfied. In matrix notation, the case may be written as:

$$X'(t) = AX(t)$$

$$\begin{pmatrix} P'(t) \\ S'(t) \\ B'(t) \\ E'(t) \\ A'(t) \end{pmatrix} = \begin{pmatrix} -\alpha & 0 & 0 & 0 & 0 \\ \beta & -\delta & 0 & 0 & 0 \\ 0 & \phi & -\rho & 0 & 0 \\ 0 & 0 & \varphi & -\eta & 0 \\ 0 & 0 & 0 & \sigma & 0 \end{pmatrix} \begin{pmatrix} P(t) \\ S(t) \\ B(t) \\ E(t) \\ A(t) \end{pmatrix}, A = \begin{pmatrix} -\alpha & 0 & 0 & 0 & 0 \\ \beta & -\delta & 0 & 0 & 0 \\ 0 & \phi & -\rho & 0 & 0 \\ 0 & 0 & \varphi & -\eta & 0 \\ 0 & 0 & 0 & \sigma & 0 \end{pmatrix} \tag{6}$$

We see that the nature of the solutions of the system will depend on the eigenvalues of the matrix. On the roots of the characteristic equation:

$$\lambda^5 + (\alpha+\delta+\rho+\eta)\lambda^4 + (\alpha\delta+\alpha\eta+\alpha\rho+\delta\eta+\delta\rho+\eta\rho)\lambda^3 + (\alpha\delta\eta+\alpha\delta\rho+\alpha\eta\rho+\delta\eta\rho)\lambda^2 + \eta\rho\delta\alpha\lambda = 0 \tag{7}$$

with eigenvalues obtained as:

$$\lambda_{i,i=1,2,3,4,5} = \begin{pmatrix} 0 \\ -\rho \\ -\eta \\ -\delta \\ -\alpha \end{pmatrix} \tag{8}$$

It can easily be seen from (8) thatall the eigenvalues are negative except one which is zero. The zero eigenvalue can lead to different stability scenarios depending on the nature of the accompanying eigenvalues. If all other eigenvalues have negative real parts, the equilibrium is a stable center (Obasi, 2023). The negative eigenvalues indicate that small perturbations from the equilibrium will decay over time, and the system will return to the balanced state. In the context of the hierarchy of needs, negative eigenvalues would imply that, when the physiological, safety, and love and belonging needs are at an equilibrium point, any deviations from this point are dampened, and the individual tends to return to a balanced state in terms of these needs. This suggests a stable and adaptive psychological state where the person's needs are generally met and maintained over time. In practical terms, this means that individuals are inclined to return to a balanced state where their physiological, safety, and social needs are met.

Stability in the system implies that when an individual's needs are satisfied at a certain level, they are more likely to maintain a state of well-being. The negative eigenvalues reflect a tendency for individuals to maintain a balanced and satisfied state in terms of their needs, fostering psychological well-being. Negative eigenvalues suggest that the system is adaptive and resilient to perturbations. In the face of external or internal challenges, individuals tend to return to a stable state. This aligns with Maslow's idea that individuals have an inherent drive for self-actualization and growth, and negative eigenvalues indicate a capacity to adapt and recover from disruptions. The hierarchy of needs is characterized by the transition from lower-level needs to higher-level needs as lower-level needs are satisfied. Individuals, having satisfied lower-level needs, are more likely to progress to addressing higher-level needs, such as esteem and self-actualization, in a stable manner. To further understand the system dynamics, exact model solution is determined. Thus, the system of equations (5) are first-order linear ODEs, which can be solved via the integrating factor method. Therefore, the exact solutions of the system of equations (5) is obtained as:

$$
\left\{
\begin{array}{l}
P(t)=P_0e^{-\alpha t},\ as\ t\to\infty,\ P(t)\to 0 \\
S(t)=\frac{P_0\beta}{\delta-\alpha}e^{(\delta-\alpha)t}+\left(S_0+\frac{P_0\beta}{\delta-\alpha}\right)e^{-\delta t},\ as\ t\to\infty,\ \mathbf{S}(t)\to 0,\ \delta<\alpha \\
B(t)=\phi e^{-\beta t}\left[\frac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)}e^{(\delta-\alpha+\beta)t}+\frac{1}{\beta-\delta}\left(S_0+\frac{P_0\beta}{\delta-\alpha}\right)e^{(\beta-\delta)t}\right]+\left[B_0-\phi\left(\frac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)}+\frac{1}{\beta-\delta}\left(S_0+\frac{P_0\beta}{\delta-\alpha}\right)\right)\right]e^{-\beta t},\ as\ t\to\infty,\ \mathbf{B}(t)\to 0 \\
E(t)=\varphi\phi\left[\frac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)(\delta-\alpha+\eta)}e^{(\delta-\alpha+\eta)t}-\frac{1}{(\beta-\delta)(\delta-\eta)}\left(S_0+\frac{P_0\beta}{\delta-\alpha}\right)e^{(\delta-\eta)t}-\frac{B_0}{\beta-\eta}e^{-(\beta-\eta)t}-\frac{\phi}{2\beta-\eta}\left(\frac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)}+\frac{1}{\beta-\delta}\left(S_0+\frac{P_0\beta}{\delta-\alpha}\right)\right)e^{-(2\beta-\eta)t}\right] \\
+\left[E_0-\varphi\phi\left(\frac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)(\delta-\alpha+\eta)}-\frac{1}{(\beta-\delta)(\delta-\eta)}\left(S_0+\frac{P_0\beta}{\delta-\alpha}\right)-\frac{B_0}{\beta-\eta}-\frac{\phi}{2\beta-\eta}\left(\frac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)}+\frac{1}{\beta-\delta}\left(S_0+\frac{P_0\beta}{\delta-\alpha}\right)\right)\right)\right] \\
as\ t\to\infty,\ \mathbf{E}(t)\to E^*,\ provided\ \delta-\alpha+\eta<0,\ \delta-\eta<0,\ \beta-\eta>0,\ 2\beta-\eta>0, \\
where \\
E^*=E_0-\varphi\phi\left(\frac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)(\delta-\alpha+\eta)}-\frac{1}{(\beta-\delta)(\delta-\eta)}\left(S_0+\frac{P_0\beta}{\delta-\alpha}\right)-\frac{B_0}{\beta-\eta}-\frac{\phi}{2\beta-\eta}\left(\frac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)}+\frac{1}{\beta-\delta}\left(S_0+\frac{P_0\beta}{\delta-\alpha}\right)\right)\right) \\
A(t)=\sigma\varphi\phi\left[\frac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)(\delta-\alpha+\eta)^2}e^{(\delta-\alpha+\eta)t}-\frac{1}{(\beta-\delta)(\delta-\eta)^2}\left(S_0+\frac{P_0\beta}{\delta-\alpha}\right)e^{(\delta-\eta)t}+\frac{B_0}{(\beta-\eta)^2}e^{-(\beta-\eta)t}+\frac{\phi}{(2\beta-\eta)^2}\left(\frac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)}+\frac{1}{\beta-\delta}\left(S_0+\frac{P_0\beta}{\delta-\alpha}\right)\right)e^{-(2\beta-\eta)t}\right] \\
+\left[E_0-\varphi\phi\left(\frac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)(\delta-\alpha+\eta)}-\frac{1}{(\beta-\delta)(\delta-\eta)}\left(S_0+\frac{P_0\beta}{\delta-\alpha}\right)-\frac{B_0}{\beta-\eta}-\frac{\phi}{2\beta-\eta}\left(\frac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)}+\frac{1}{\beta-\delta}\left(S_0+\frac{P_0\beta}{\delta-\alpha}\right)\right)\right)\right]t \\
+A_0-\sigma\varphi\phi\left[\frac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)(\delta-\alpha+\eta)^2}-\frac{1}{(\beta-\delta)(\delta-\eta)^2}\left(S_0+\frac{P_0\beta}{\delta-\alpha}\right)+\frac{B_0}{(\beta-\eta)^2}+\frac{\phi}{(2\beta-\eta)^2}\left(\frac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)}+\frac{1}{\beta-\delta}\left(S_0+\frac{P_0\beta}{\delta-\alpha}\right)\right)\right] \\
as\ t\to\infty,\ \mathbf{A}(t)\to A^*,\ provided\ \delta-\alpha+\eta<0,\ \delta-\eta<0,\ \beta-\eta>0,\ 2\beta-\eta>0, \\
where \\
A^*=A_0-\sigma\varphi\phi\left[\frac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)(\delta-\alpha+\eta)^2}-\frac{1}{(\beta-\delta)(\delta-\eta)^2}\left(S_0+\frac{P_0\beta}{\delta-\alpha}\right)+\frac{B_0}{(\beta-\eta)^2}+\frac{\phi}{(2\beta-\eta)^2}\left(\frac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)}+\frac{1}{\beta-\delta}\left(S_0+\frac{P_0\beta}{\delta-\alpha}\right)\right)\right]
\end{array}
\right.
$$

From the exact solutions, the tendency of physiological, safety, belongingness and love, esteem, and self-actualization needs to approach zero or equilibrium values indicates a long-term stability in the system. Individuals in the model reach a balanced state where their various needs, spanning from basic survival to higher-level self-fulfillment, are either completely satisfied or in a state of sustained equilibrium. The result suggests that, over time, individuals progress through Maslow's hierarchy of needs and eventually reach a state where the pursuit of these needs becomes less dominant. Basic physiological and safety needs tend to zero, indicating that individuals achieve a stable state of physical well-being and security. Meanwhile, higher-level needs like belongingness, esteem, and self-actualization tend to equilibrium values, suggesting a sustained and balanced pursuit of social connection, recognition, and personal growth.

The findings imply that individuals in the model adapt to their environments and find satisfaction in the fulfillment of their diverse needs. As basic needs are met, the focus shifts to higher-level needs, leading to a dynamic and evolving psychological state. The tendency of self-actualization needs to approach equilibrium values indicates that individuals, even in a state of equilibrium, continue to experience personal growth and pursue self-fulfillment. This aligns with Maslow's concept of self-actualization as an ongoing process of realizing one's potential. This goes to show that individuals have a natural inclination to maintain a stable and satisfied state, adapting to changing circumstances while progressing through the hierarchy of needs. This stability aligns with the notion that individuals strive for fulfillment and self-actualization in a resilient and balanced manner. The result suggests a long-term stability and adaptation in the modeled hierarchy of needs. Individuals reach a balanced state where basic needs are satisfied, and higher-level needs are pursued in a sustainable and evolving manner. The exact solutions highlight the dynamic and continuous nature of human motivation and well-being as individuals progress through the hierarchy over an extended period.

Social Learning Theory

Social learning theory (SLT), developed by psychologist Albert Bandura (Bandura, 1977), is a comprehensive theory that emphasizes the importance of social interactions in learning. It proposes that individuals learn not only through direct experiences but also by observing others and modeling their behaviours, attitudes, and emotional reactions. Bandura's work expanded traditional behaviourist theories by integrating cognitive and social aspects into the learning process. The key concepts of social learning theory include: observational learning, modeling, reinforcement, attention, retention, reproduction, motivation, and self-efficacy (Bandura, 1986). Observational learning also known as modeling or vicarious learning occurs when individuals observe the actions and outcomes of others and incorporate this information into their own behaviours. Bandura's famous Bobo doll experiment demonstrated how children learned aggressive behaviours by observing adults. Modeling involves imitating the behaviours of role models or significant others. Individuals are more likely to emulate behaviours they perceive as rewarding or socially acceptable. For example, children may imitate the behaviours of parents, teachers, or peers. While traditional behaviourist theories emphasize external reinforcement, SLT suggests that reinforcement can be vicarious (Bandura & Walters, 1977). Individuals are motivated to imitate behaviours that result in positive outcomes for others. Conversely, observing negative consequences for certain behaviours may inhibit imitation. Bandura proposed a four-step process involved in observational learning. Individuals must pay attention to the model's behaviour and its consequences. They must remember the observed behaviour. Individuals must have the ability to reproduce the observed behaviour. Finally, individuals are more likely to imitate behaviours if they are motivated to do so, either through intrinsic or extrinsic factors. Central to social learning theory is the concept of self-efficacy, which refers to an individual's belief in their own ability to succeed in specific situations or accomplish tasks (Bandura, 2001). Bandura argued that self-efficacy influences motivation, behaviour, and resilience in the face of challenges.

Representing social learning theory using mathematics offers several advantages. Mathematical models provide a systematic framework for quantitatively analyzing the dynamics of social learning. By expressing SLT concepts in mathematical terms, researchers can conduct rigorous analyses, perform simulations, and make predictions about learning outcomes. Mathematics enables clear and precise formulation of SLT concepts, making them easier to understand and communicate. Mathematical models can formalize complex relationships between variables, parameters, and processes involved in social learning. Mathematical modeling allows researchers to dissect and understand the underlying mechanisms driving social learning. By representing SLT in mathematical terms, researchers can identify key factors influencing learning outcomes and explore how they interact. Well-constructed mathematical models of SLT can yield predictive insights into learning behaviours and outcomes. Researchers can use simulations to explore hypothetical scenarios and assess the potential impact of interventions or changes in social contexts. In the words of Fadul (2006), mathematics serves as a common language across disciplines, facilitating interdisciplinary integration of SLT with fields such as psychology, sociology, education, and computer science. Mathematical modeling encourages collaboration and exchange of ideas among researchers from diverse backgrounds. Mathematical models of SLT can inform the design of educational interventions and instructional strategies. By understanding the underlying mechanisms of social learning, educators can develop more effective approaches to teaching and fostering collaborative learning environments.

By leveraging mathematical tools and techniques, learning theorists can develop precise models, formulate hypotheses, conduct empirical studies, and derive insights into the mechanisms underlying learning processes and behaviour. Mathematics provides a rigorous framework for studying and understanding the complexities of learning theory, facilitating advancements in educational research and practice (Fadul, 2006). Therefore, representing social learning theory using mathematics enhances our understanding of the complex interplay between social interactions, cognitive processes, and learning outcomes. Mathematical modeling provides a powerful tool for exploring, analyzing, and advancing our knowledge of social learning phenomena. To develop a system of differential equations describing social learning theory, we need to consider the key components of this theory, which emphasize learning through observation, modeling, and social interaction. The model variables and parameters are defined in Table 2.

Table 2. Model variable and parameter descriptions

Variables	Descriptions
L(t)	Level of learning or knowledge at time
I(t)	Intensity of exposure to social interactions or observational learning at time.
Parameters	
α	Rate at which learning occurs through observation.
β	Rate at which learning occurs through direct experience or instruction.
γ	Rate at which forgetting occurs.
σ	Influence of reinforcement or feedback.

Based on these variables and parameters, we can formulate a system of differential equations describing the social learning theory as developed Albert Bandura:

$$\frac{L(t)}{dt} = \alpha I(t) - \beta L(t) - \gamma L(t) \qquad (9)$$

This equation describes how the level of learning (*L(t)*) changes over time. Learning increases based on the intensity of exposure to social interactions (*I(t)*) with a rate proportional to *α*. Learning also decreases over time due to forgetting (*γL(t)*) and through direct experience or instruction (*βL(t)*). Additionally, we can introduce a differential equation to model the intensity of exposure to social interactions (*I(t)*):

$$\frac{I(t)}{dt} = \delta\left(L(t) - I(t)\right) \qquad (10)$$

This equation describes how the intensity of exposure to social interactions changes over time. It depends on the difference between the level of learning (*L(t)*) and the current intensity of exposure to social interactions (*I(t)*), with α rate proportional to *σ*. Thus, putting the two equations together we have

$$\begin{cases} \frac{L(t)}{dt} = \alpha I(t) - \beta L(t) - \gamma L(t) \\ \frac{I(t)}{dt} = \delta\left(L(t) - I(t)\right) \end{cases} \qquad (11)$$

This system of differential equations captures the essence of social learning theory by modeling how learning evolves over time through observation, direct experience, forgetting, and feedback from social interactions. Adjustments and refinements can be made to this model to account for specific contexts and factors influencing social learning. In matrix notation, the case may be written as:

$$\begin{aligned} & X'(t) = AX(t) \\ & \begin{pmatrix} L'(t) \\ I'(t) \end{pmatrix} = \begin{pmatrix} -(\beta+\gamma) & \alpha \\ \delta & -\delta \end{pmatrix} \begin{pmatrix} L(t) \\ I(t) \end{pmatrix} \end{aligned} \qquad (12)$$

We see that the nature of the solutions of the system will depend on the eigenvalues of the matrix. The determinant, and trace, of this matrix play a crucial role in determining the stability of the solutions. If and , the system is stable. These criteria are derived from the fact that the signs of the eigenvalues influence the stability of the system (Obasi & Mbah, 2019; Obasi, 2023). Positive real parts of eigenvalues are associated with instability, while negative real parts suggest stability. The determinant and trace help in identifying these signs and making predictions about the system's behaviour. On the roots of the characteristic equation:

$$\lambda^2 + \left(\delta + \beta + \gamma\right)\lambda - \delta\alpha + \delta\beta + \delta\gamma = 0 \qquad (13)$$

with the determinant and trace of matrix *A* obtained as:

$$\begin{cases} \det(A) = -\delta\alpha + \delta\beta + \delta\gamma \\ tr(A) = -\left(\delta + \beta + \gamma\right) \end{cases} \qquad (14)$$

It can easily be seen from (14) that, $tr(A) < 0$, but if $\beta + \gamma > \alpha$ then $det(A) > 0$, which implies stability. That is, if the sum of rate at which learning occurs through direct experience or instruction and the rate at which forgetting occurs is greater than the rate at which learning occurs through observation, there will be increased learning. In the context of social learning theory, stability in the system of differential equations implies that the behaviours or patterns modeled by the equations tend to settle down or remain consistent over time. When the system is stable, it suggests that the dynamics of learning and behaviour tend to reach an equilibrium or a steady state. In other words, the behaviours modeled by the equations don't wildly fluctuate or diverge from each other over time. Instead, they converge towards certain patterns or norms, reflecting the stability of the social learning process within the theoretical framework being studied. This finding suggests that the effectiveness of learning can be enhanced when the combined rate of learning from direct experience or instruction, along with the rate of forgetting, surpasses the rate of learning through observation. This underscores the importance of active learning approaches, such as hands-on practice, interactive instruction, and spaced repetition, in promoting deep understanding and long-term retention of knowledge and skills. By balancing direct experience with effective instructional techniques and reinforcement strategies, educators and learners can optimize the learning process and enhance educational outcomes.

The stability of social learning theory in the context of learning has several important implications. Stable social learning processes imply that individuals are likely to consistently adopt and exhibit behaviours that are learned from observing others in their social environment. This consistency can lead to the establishment and maintenance of social norms and cultural practices within a group or society. When social learning processes are stable, it becomes easier to predict how individuals will respond to different social cues or stimuli. This predictability can be valuable for educators, policymakers, and social scientists who seek to understand and influence behaviour within various social contexts. It facilitates the transmission of knowledge, skills, and cultural traditions across generations. When individuals reliably learn from their social environment, valuable information and practices can be passed down from one generation to the next, contributing to the continuity and evolution of society. More so, it can contribute to social cohesion by promoting shared values, beliefs, and behaviours within a group or community. When individuals consistently learn from and imitate each other, it fosters a sense of belonging and solidarity, which can strengthen social bonds and cooperation. While stability implies consistency, it does not necessarily mean rigidity. Social learning processes can also be adaptive, allowing individuals and societies to respond to changing circumstances and environments. Stable social learning frameworks provide a foundation upon which new behaviours and innovations can be integrated and disseminated. A stable social learning system can enhance the resilience of individuals and communities by providing a reliable mechanism for acquiring knowledge and coping strategies in the face of challenges or adversity. Consistent learning from others can help individuals navigate uncertain or unfamiliar situations more effectively. Thus, the stability of social learning theory in learning underscores its significance as a fundamental mechanism through which individuals acquire knowledge, skills, and social behaviours within their cultural and social contexts. To further understand the system dynamics, exact model solution is determined. The system of equations (11) are first-order linear ODEs, which can be solved via the matrix method. We have that

$$X = e^{At}X(0),\ X(0) = \begin{pmatrix} L_0 \\ I_0 \end{pmatrix}$$

$$\begin{cases} \dfrac{dX(t)}{dt} = AX(t) = \begin{pmatrix} -(\beta+\gamma) & \alpha \\ \delta & -\delta \end{pmatrix}\begin{pmatrix} L(t) \\ I(t) \end{pmatrix},\ X(t) = e^{At}\begin{pmatrix} L_0 \\ I_0 \end{pmatrix},\ but \\ e^{At} = 1 + At + \dfrac{A^2}{2!}t^2 + \dfrac{A^3}{3!}t^3 + \dots \\ \therefore\ e^{At} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -(\beta+\gamma) & \alpha \\ \delta & -\delta \end{pmatrix}t + \dfrac{1}{2!}\begin{pmatrix} -(\beta+\gamma) & \alpha \\ \delta & -\delta \end{pmatrix}^2 t^2 + \dfrac{1}{3!}\begin{pmatrix} -(\beta+\gamma) & \alpha \\ \delta & -\delta \end{pmatrix}^3 t^3 + \dots \\ = \begin{pmatrix} 1-(\beta+\lambda)t+\frac{1}{2}\left((\beta+\gamma)^2+\delta\alpha\right)t^2 & \alpha t-\frac{1}{2}(\beta+\gamma-\delta)\alpha t^2 \\ \delta t-\frac{1}{2}(\beta+\gamma-\delta)\delta t^2 & 1-\delta t+\frac{1}{2}(\alpha+\delta)\delta t^2 \end{pmatrix} \end{cases}$$

$$\begin{cases} X = \begin{pmatrix} 1-(\beta+\lambda)t+\frac{1}{2}\left((\beta+\gamma)^2+\delta\alpha\right)t^2 & \alpha t-\frac{1}{2}(\beta+\gamma-\delta)\alpha t^2 \\ \delta t-\frac{1}{2}(\beta+\gamma-\delta)\delta t^2 & 1-\delta t+\frac{1}{2}(\alpha+\delta)\delta t^2 \end{pmatrix}\begin{pmatrix} L_0 \\ I_0 \end{pmatrix} \\ X(t) \equiv (L(t), I(t)) = \begin{pmatrix} L_0-(\beta+\lambda)L_0 t+\frac{1}{2}L_0\left((\beta+\gamma)^2+\delta\alpha\right)t^2+I_0\alpha t-\frac{1}{2}I_0(\beta+\gamma-\delta)\alpha t^2 \\ L_0\delta t-\frac{1}{2}L_0(\beta+\gamma-\delta)\delta t^2+I_0(1-\delta t)+\frac{1}{2}I_0(\alpha+\delta)\delta t^2 \end{pmatrix} \end{cases}$$

Therefore, the solution of the system of equations (11) is obtained as:

$$\begin{cases} L(t) = L_0-(\beta+\lambda)L_0 t+\frac{1}{2}L_0\left((\beta+\gamma)^2+\delta\alpha\right)t^2+I_0\alpha t-\frac{1}{2}I_0(\beta+\gamma-\delta)\alpha t^2+O(t^3) \\ I(t) = L_0\delta t-\frac{1}{2}L_0(\beta+\gamma-\delta)\delta t^2+I_0(1-\delta t)+\frac{1}{2}I_0(\alpha+\delta)\delta t^2+O(t^3) \end{cases} \quad (15)$$

$$as\ t\to\infty, \boldsymbol{L}(t)\to\infty, \boldsymbol{I}(t)\to\infty$$

The exact solutions (15) above show the idea that level of learning or knowledge and the intensity of exposure to social interactions or observational learning could theoretically approach infinity as time tends to infinity. This suggests that, theoretically, there is no upper limit to the level of learning or knowledge that individuals can attain, nor to the intensity of exposure to social interactions or observational learning, given an infinite amount of time. Level of learning or knowledge refers to the depth and breadth of understanding that individuals can achieve in various domains, such as academic subjects, skills, or personal development. The finding suggests that as individuals engage in learning activities over an extended period, their level of knowledge can continue to grow indefinitely. This implies that there is always more to learn, discover, and understand, and that the process of learning is potentially limitless. It goes to show that social interactions and observational learning play crucial roles in the learning process. Through interactions with others, individuals can exchange ideas, perspectives, and information, which can enrich their understanding and stimulate further learning. Observational learning, where individuals learn by observing and imitating others, also contributes significantly to the acquisition of new skills and behaviours. The finding suggests that as individuals engage in more social interactions and observational learning experiences over time, the potential for learning increases without bounds.

Therefore, given an infinite amount of time, there is no limit to how much individuals can learn or how intensively they can engage in social interactions or observational learning. In other words, as time goes on indefinitely, the potential for learning and exposure to learning opportunities continues to grow without constraint. However, it is essential to note that while this finding provides an intriguing theoretical perspective on the potential for lifelong learning and continuous personal growth, it is practically impossible for individuals to have infinite time or resources for learning. In reality, people have finite lifespans, limited resources, and various constraints that affect their learning opportunities. Nonetheless, the idea that learning and exposure to learning experiences can continue to expand without limit underscores the importance of fostering a lifelong learning mindset and seeking out opportunities for growth and development throughout one's life.

Concluding Remarks

This paper delves into the intersection of mathematical modeling and learning theory, exploring how mathematical frameworks have been instrumental in advancing our understanding of educational processes and pedagogical practices. Advancing learning theories through mathematical modeling offers several significant benefits and opportunities. This is because mathematical models provide a precise and systematic framework for representing complex educational phenomena, allowing researchers to formalize theoretical concepts, relationships, and mechanisms in a clear and rigorous manner. This paper focused on such learning theories as cooperative learning theory, Maslow motivation theory, and social learning theory. By employing mathematical models, researchers have been able to dissect complex educational phenomena, elucidate underlying mechanisms, and predict outcomes with a level of precision that traditional qualitative approaches often struggle to achieve.

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Динаміка математичної моделі деяких теорій навчання на основі штучного інтелекту

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Авторський вклад: A – дизайн дослідження; B – збір даних; C – statаналіз; D – підготовка рукопису; E – збір коштів

Реферат. Стаття: 5 с., 4 табл., 36 джерел.

Мета дослідження. У цій статті розглянутий взаємозв'язок математичного моделювання та теорії навчання, досліджено, як математичні рамки відіграли важливу роль у просуванні нашого розуміння освітніх процесів і педагогічних практик за допомогою штучного інтелекту (ШІ).

Матеріал і методи. Розвиток теорій навчання за допомогою математичного моделювання пропонує кілька значних переваг і можливостей. Це пов'язано з тим, що математичні моделі забезпечують точну та систематичну основу для представлення складних освітніх явищ, дозволяючи дослідникам формалізувати теоретичні концепції, взаємозв'язки та механізми у чіткий та строгий спосіб.

Результати та висновки. Ця стаття була присвячена таким теоріям навчання, як теорія кооперативного навчання, теорія мотивації Маслоу та теорія соціального навчання. Використовуючи математичні моделі, були проаналізовані складні освітні явища, з'ясовані основні механізми та передбачині результати з таким рівнем точності, якого часто важко досягти традиційними якісними підходами.

Ключові слова: навчання, теорія навчання, кооператив, мотивація, теорія соціального навчання.

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